AI-1553

M.A./M.Sc. (Final) Mathematics

Term End Examination, 2020-21

Compulsory/Optional

Group-

Paper-

FUZZY SETS AND THEIR APPLICATIONS

Time:- Three Hours]

[Maximum Marks:100

[Minimum Passing Marks: 036

Note: Answer any five question. All question carry equal marks.

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1. (a) Define convex fuzzy set and show that a fuzzy set A on R is convex iff $A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)]$ for all $x_1, x_2 \in$ R **10** and all $\lambda \in [0,1]$

   (b) Define standard complement of a fuzzy set, standard union and standard intersection of two fuzzy sets. **10**

   If fuzzy sets A_1, A_2 defined on $[0,80] = X$ by

$$A_1(x) = \begin{cases} 1 & x \leq 20 \\ \frac{35-x}{15} & 20 < x < 35 \\ 0 & x \geq 35 \end{cases},$$

$$A_1(x) = \begin{Bmatrix} 0 \quad when\ either\ x \leq 20\ or\ \geq 60 \\ \dfrac{x-20}{15} \quad 20 < x < 35 \\ \dfrac{60-x}{15} \quad 45 < x < 60 \\ 1 \quad 35 \leq x \leq 45 \end{Bmatrix}$$

   The find $\overline{A_1}$, $A_1 \cup A_2$ and $A_1 \cap A_2$

2. (a) Show that the standard fuzzy union of infinite sets is strong cutworthy but not cutworthy. **10**

   (b) Let $f: X \to Y$ be an arbitrary crisp function $A_i \in F(x)$ and $B_i \in F(y), i \in I$. Then show that the following properties of functions obtained by the extension principle hold:**10**

   (i) If $A_1 \subseteq A_2$ then $f(A_1) \subseteq f(A_2)$

   (ii) $f(\bigcap_{i \in I} Ai) \subseteq \bigcap_{i \in I} f(A_i)$

   (iii) If $B_1 \subseteq B_2$ then $f^{-1}(B_1) \subseteq f^{-1}(B_2)$

3. (a) If C is a continuous fuzzy complement the show that c has a unique equilibrium.**10**

   (b) Let f be a decreasing generator. The show that a function g defined by **10**

$$g(a) = f(0) - f(a) \qquad for \ \ a \in [0,1]$$

   Is an increasing generator with $g(1) = f(0)$and its pseudo inverse$g^{(-1)}$ is given by $g^{(-1)}(a) = f^{(-1)}(f(0) - a) \qquad for \ \ a \in R.$

4. State and prove First characterization theorem of fuzzy complements. **20**

5. (a) Write short notes on Lattice of fuzzy numbers. **10**

(b) Write short notes on fuzzy equations. **10**

6. (a) Define fuzzy equivalence relation with an example. **10**

   (b) For $a, b, d, a_j \in [0,1]$ where $j \in J$. Show that **10**

   (i) $i(a, b) \leq d \quad iff\ w_i(a, d) \geq b$

   (ii) $w_i \left[\sup_{i \in I} a_j, b \right] = \inf_{i \in I} w_i(a_j, b)$

7. (a) If a finite body of evidence (F,m) be nested. Then show that – **10**

   (i) $Bel(A \cap B) = \min[BelA, BelB]$

   (ii) $Pl\,(A \cup B) = \max[Pl(A), Pl(B)]$ for all $A, B \in P(x)$.

   (b) Show that a belief measure Bel on a finite power set P(x) is a probability measure if and only if the associated basic probability assignment function m is given by **10**

$$m(\{x\}) = Bel\,(\{x\}) and\ m(A) = 0$$
$$for\ all\ subsets\ of\ X\ that\ are\ not\ singletons.$$

8. Write an essay on fuzzy propositions. **20**

9. (a) Let A be a normal fuzzy set. For any continuous t-norm I and associated w_i operator. If $\tau = w_i$. **10**

   That is $\tau\big(A(x), B(y)\big) = w_i\big(A(x), B(y)\big) \quad \forall x \in X, y \in Y$

   Then show that $B(y) = \sup_{x \in X} i[A(x), \tau(A(x), B(y)]$

   (b) In multiconditional approximate reasoning, there are four possible ways of calculating the conclusion B' **10**

$$B'_1 = A'0(\bigcup_{j \in N_n} R_j)$$
$$B'_2 = A'0(\bigcup_{j \in N_n} R_j)$$
$$B'_3 = \bigcup_{j \in N_n} A'0R_j$$
$$B'_4 = \bigcup_{j \in N_n} A'0R_j$$

   Then show that $B'_2 \subseteq B'_4 \subseteq B'_1 = B'_3$

10. (a) Write short notes on defuzzification methods. **10**

    (b) Write short on Fuzzy Automata. **10**