

**AI-1550**  
**M.A./M.Sc. (Final) Mathematics**  
**Examination- MAR-APR 2021**  
**Compulsory/Optional**  
**Paper-VI**  
**Paper Title: Fluid Mechanics**

**Time:- Three Hours ]**

**[Maximum Marks:100**  
**[Minimum Marks: 036**

**Note: Answer any five questions. All question carry equal marks.**

1. Determine the acceleration at the point (2,1,3) at  $t=0.5$ sec. If  $u = yz + t$ ,  $v = xz + t$  and  $w = xy$ .
2. A mass of fluid is in motion so that the lines of motion lie on the surface of coaxial cylinder, show that the equation of continuity is  $\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(\rho U_{\theta})}{\partial \theta} + \frac{\partial(\rho V_z)}{\partial z} = 0$  where  $V_{\theta}, V_z$ , are the velocities perpendicular and parallel to  $Z$ .
3. If the velocity of an incompressible fluid at the point (x,y,z) is given by  $\left(\frac{3xy}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right)$  prove that the liquid motion is possible and the velocity potential is  $\frac{\cos \theta}{r^2}$ .
4. Show that the ellipsoid  $\frac{x^2}{a^2 k^2 t^{2n}} + k t^n \left(\frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 1$  is a possible form of the boundary surface of a liquid.
5. Show that  $\int \frac{dp}{\rho} + \frac{1}{2} q^2 + V = C$  where the motion is steady and the velocity potential does not exist,  $V$  being the potential from which the external forces are derivable.
6. Air obeying Boyle's law is in motion in a uniform tube of small section, prove that if  $e$  be the density and  $v$  the velocity at a distance  $x$  from a fixed point at time  $t$   $\frac{\partial^2 e}{\partial t^2} - \frac{\partial^2}{\partial x^2} \{(V^2 + k)e\}$
7. Find complex Potential for a Source.
8. Use the method of images to prove that if there be a surface  $m$  at point in a fluid bounded by the lines  $\theta = 0$  and  $\theta = \frac{1}{3}\pi$  the solution is  $\phi + i\psi = -m \log\{(z^3 - z_0^3)(z^3 - z_0'^3)\}$  where  $z_0 = x_0 + iy_0$  and  $z_0' = x_0 + iy_0$ .
9. A sphere of radius  $a$  is moving with constant velocity  $U$  through an infinite liquid at rest at infinity. If  $P_0$  be the pressure at infinity show that the pressure at any point of the surface of the sphere, the radius to which point makes an angle  $\theta$  with the direction of motion is given by  $P = P_0 + \frac{1}{2} \rho U^2 \left(1 + \frac{9}{4}\right) \sin^2 \theta$ .
10. Show that the flux of velocity through any cross-section of a vortex tube is a constant all along the tube.