AI-1545

M.A./M.Sc. (Final) Mathematics

Term End Examination, 2020-21

Compulsory/Optional

Group-

Paper-

INTEGRATION THEORY & FUNCTIONAL ANALYSIS

Time:- Three Hours] [Maximum Marks:100

[Minimum Passing Marks:036

Note: Answer any five questions. All questions carry equal marks.

1. State and prove Hahn decomposition theorem.
2. State and prove Riesz Representation theorem.
3. Let P be a real number such that is $1 \leq P < \infty$ and l_p^n denote the linear space of all n-tuples of scalars with the norm of a vector $x = (x_1, x_2 \ldots\ldots x_n)$defined by $\|x\|_p = (\sum |xi|^p)^{1/p}$ Show that l_p^n is a Banach space.
4. Let T be a linear transformation of a normed linear space N into another normed Linear space N'. Then the following statements are equivalent-
   (i) T is continuous
   (ii) T is continuous at the origin in the sense that $x_n \to 0 \Leftrightarrow T(x_n) \to 0$
   (iii) There exists a real number $k \geq 0$ such that $\|T(x)\| \leq k\|x\|, \forall x \in N$.
5. State and prove closed graph theorem.
6. State and prove uniform boundedness theorem.
7. (a) Prove that a Banach space is Hilbert space if and only if the Parallelogram law hold.
   (b) State and prove Bessel's Inequality.
8. Let Y be a fixed vector in a Hilbert space it and let f_y be a scalar valued function on H defined $f_y\,(x) = (x, y), \forall\, x \in H$
9. Let T be on operator on a Hilbert space H then there exist a unique operator T* on H such that (Tx,y)=(x,T*y) $\forall x, y \in H$
10. (a) If T is an operator an a Hilbert space H, then (Tx,x)=0 for all x in H⇔T=0
    (b) If T is a positive operator an a Hilbert space H. Then I+T is non-singular.